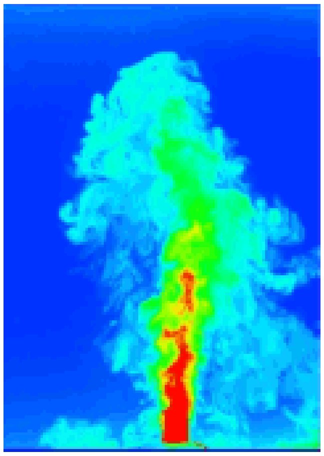

Journal of Turbulence
Publication details, including instructions for authors and subscription information:
http://www.informaworld.com/smpp/title~content=t713665472

Reynolds number dependence of Lagrangian statistics in large numerical simulations of isotropic turbulence

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First Published on: 01 January 2006
To cite this Article: Yeung, P. K., Pope, S. B. and Sawford, B. L. (2006) 'Reynolds number dependence of Lagrangian statistics in large numerical simulations of isotropic turbulence', Journal of Turbulence, Volume 7, Art. No. N58,
To link to this article: DOI: 10.1080/14685240600868272
URL: http://dx.doi.org/10.1080/14685240600868272

Journal of Turbulence
Volume 7, No. 58, 2006

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Reynolds number dependence of Lagrangian statistics in large numerical simulations of isotropic turbulence

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Lagrangian statistics are reported from a direct numerical simulation database with grid resolution up to 2048^3 and Taylor-scale Reynolds number approximately 650. The approach to Lagrangian Kolmogorov similarity at high Reynolds number is studied using both the velocity structure function and frequency spectrum. A significant scaling range is observed for the latter which is consistent with recent estimates of 6–7 for the scaling constant C_0. In contrast to some previous results at low Reynolds number, the current results suggest that at high Reynolds number the dissipation autocorrelation is a two-scale process influenced by both the Lagrangian velocity integral time scale and Kolmogorov time scale. Results on the logarithm of the pseudo-dissipation are in support of its modeling as a diffusion process with one-time Gaussian statistics. As the Reynolds number increases, the statistics of dissipation and enstrophy become more similar while their logarithms have significantly longer time scales.

1. Introduction

The study of turbulence from a Lagrangian viewpoint has a long history, with the well-known works of Taylor [1] and Richardson [2] both pre-dating Kolmogorov [3] whose hypotheses of small-scale universality at high Reynolds number are extremely important in the field. However, understanding of the high-Reynolds-number behaviour of Lagrangian statistics has lagged behind that for Eulerian spatial properties often used to characterize turbulence at the small scales [4]. In particular, application of Kolmogorov's inertial-range similarity to Lagrangian statistics in time is still uncertain, and known (e.g. Yeung [5]) to require higher Reynolds numbers. A principal reason for these difficulties is that the range of time scales in the Lagrangian description is generally more limited, and increases less rapidly with the Reynolds number, than the range of length scales for Eulerian quantities. This makes it necessary, both in laboratory experiments [6, 7] and computations (e.g. [8], and this paper) to strive toward higher Reynolds numbers in building a database which makes a systematic study of Reynolds number dependence possible. Recently, advances in both experiment and computation have helped stimulate much interest in topics such as the scaling properties of high-order Lagrangian structure functions [9] and acceleration intermittency (e.g. [10, 11]) which are characteristic of turbulence at high Reynolds number.

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Journal of Turbulence
ISSN: 1468-5248 (online only) © 2006 Taylor & Francis
http://www.tandf.co.uk/journals
DOI: 10.1080/14685240600868272

Direct numerical simulations (DNS) with particle tracking [12, 13] are a well-accepted tool for extracting Lagrangian statistics, especially when detailed and quantitative information of the type needed in stochastic modeling [14] is desired. Currently, advances in supercomputer power at the Terascale level are allowing higher Reynolds numbers in the simulations and hence new opportunities to address important scaling issues more conclusively than before. One example of fundamental importance is the so-called Lagrangian Kolmogorov constant (C_0) in the inertial-range prediction for the second-order structure function (mean-square of the Lagrangian velocity increment over a time interval τ). The prediction is

$$D_2^L(\tau) = C_0\langle\epsilon\rangle\tau, \quad (\tau_\eta \ll \tau \ll T_L), \tag{1}$$

where $\langle\epsilon\rangle$ is the mean energy dissipation, τ_η is the Kolmogorov time scale and T_L is the Lagrangian integral time scale (of the velocity). Knowledge of C_0 is very important in modeling because it controls the magnitude of T_L in stochastic models (e.g. see [15]). A convincing inference of C_0 requires a significant scaling range which would appear as a plateau in a plot of $D_2^L(\tau)/(\langle\epsilon\rangle\tau)$. The degree of uncertainty in C_0 in the literature (e.g. [16]) is much greater than that for Eulerian versions of the Kolmogorov constant such as those for the longitudinal energy spectrum ($C_K \approx 0.53$, [17]) or spatial structure function (where $C_2 = 4.02C_K$). Higher Reynolds number data under well-controlled numerical or laboratory conditions are needed to establish the asymptotic behaviour of C_0 with increasing Reynolds number and to test estimates and parameterizations given in the literature. While questions have been raised [18, 19] concerning possible effects of anisotropy and inhomogeneity on the universality of C_0, it is important to resolve the issue of high Reynolds number asymptotic behaviour first. This task is best carried out by considering the simplified case of forced, stationary isotropic turbulence.

In this paper we examine several issues using the latest data from simulations at grid resolution up to 2048^3. Our simulation database covers Taylor-scale Reynolds numbers (R_λ) from about 40 to slightly below 700, which is probably the highest Reynolds number to date for Lagrangian statistics from DNS. Our first objective is to study the issue of C_0 as outlined above, in terms of both the Lagrangian structure function and the Lagrangian frequency spectrum $E^L(\omega)$ which may also approach an inertial range with a scaling constant that can be related to C_0. The results are quite consistent with recent estimates ([20, 21]) for this flow. A second objective is to re-examine the scaling of Lagrangian autocorrelations and integral time scales for several quantities representing local relative motion in the flow. In particular, we investigate fluctuations of the energy dissipation rate ($\epsilon \equiv 2\nu s_{ij}s_{ij}$), enstrophy ($\zeta \equiv \nu\omega_i\omega_i$), and pseudo-dissipation ($\varphi \equiv \upsilon(\partial u_i/\partial x_j)^2$), where in these definitions ν, s_{ij}, ω_i and $\partial u_i/\partial x_j$ represent kinematic viscosity, strain-rate, vorticity and the full velocity gradient tensor respectively. It is well known that in homogeneous turbulence the quantities ϵ, ζ and φ have the same one-time mean values, but their higher-order moments and two-time statistics (e.g. autocorrelations) may differ. In earlier work [13] at low Reynolds number (R_λ 38–93) the integral time scales of ϵ, ζ and φ were found to be comparable to that for the velocity (i.e., T_L), with enstrophy being correlated significantly longer than the others. However, both of these trends appeared to be weaker in later work [21] when the Reynolds number was increased. In this paper we show conclusively that the high Reynolds number behavior is different, with the Kolmogorov time scale playing a greater role. Our results are complementary to those in a recent Eulerian study [11] and are expected to be useful for new efforts on incorporating intermittency effects in stochastic modeling based on the acceleration [22, 23]. From a modeling perspective our main interest is in the statistics of ϵ, ζ and φ all considered separately. However, their joint statistics such as the dissipation-enstrophy cross-correlation [13] also contain interesting information and will be reported in a subsequent paper.

In the following we first provide a brief description of our simulation database (section 2). Two separate sections (3, 4) are then devoted to the two objectives stated above: namely to study the Reynolds number dependence of (i) the velocity structure function and frequency spectrum and (ii) autocorrelations and integral time scales of dissipation rate (ϵ) and the related quantities ζ and φ. Conclusions are summarised in section 5.

2. The simulation database

The numerical simulation and post-processing algorithms employed here are essentially the same as described in previous papers (e.g. [13, 21]). We use the well-known Fourier-spectral algorithm of Rogallo [24] in a parallel code adapted to the use of as many as 2048 processors. Stationary homogeneous isotropic turbulence is obtained by stochastic forcing at the large scales using the method of Eswaran and Pope [25] where a forcing term of finite time scale (which ensures differentiability in time) is added to the Navier–Stokes equation in Fourier space. Fluid particles are tracked in the flow with their velocities obtained by cubic spline interpolation [12] which is fourth-order accurate and twice-differentiable. The latter property ensures that fluid particle velocities calculated at successive time instants and at positions a short distance apart are differentiable, such that the acceleration can be readily obtained from the velocity time series by simple finite difference in time. With adequate resolution in space the same interpolation technique can be used to obtain velocity gradients following fluid particle trajectories, and hence Lagrangian statistics of the quantities ϵ, ζ and φ which are considered in section 4.

Table 1 lists some basic parameters of our simulations including the range of physical length and time scales in the flow, based on the longitudinal integral length scale (L_1), Kolmogorov length and time scales (η and τ_η), the large-eddy turnover time ($T_E \equiv L_1/u'$ using rms velocity u') and the Lagrangian integral time scale (T_L) obtained from the velocity autocorrelation. Some of these quantities were also reported recently in [11] but differ slightly here because of statistical sampling. For better consistency we have modified the forcing parameters for 64^3 and 128^3 runs from those of previous work [13] such that all simulations listed now have the same forcing amplitudes with different viscosities (ν) as the only direct cause of differences in Reynolds number. It is clear that the range of length scales present is much wider than that of time scales, such that an inertial range in the Eulerian energy spectrum is quite well captured [26, 27]. The ratio of Lagrangian to Eulerian large-eddy time scales, i.e. T_L/T_E may be regarded as roughly constant although this depends on the large scales which are forced and thus may be flow-dependent.

Information in table 1 includes three numerical parameters for which practical choices have to be made based on the amount of CPU resources available. First is the non-dimensional

Table 1. Basic simulation parameters as discussed in Sec. 2.

N	64	128	256	512	1024	2048
R_λ	43	86	140	235	393	648
ν	0.025	0.0071	0.0028	0.0011	0.000 437	0.000 1732
$\langle\epsilon\rangle$	1.31	1.17	1.17	1.20	1.26	1.10
L_1/η	24	52	98	201	450	732
T_L/τ_η	5.4	8.6	13.1	19.8	31.1	43.8
T_L/T_E	0.720	0.770	0.802	0.763	0.697	0.763
$k_{\max}\eta$	1.77	1.41	1.41	1.40	1.37	1.44
T/T_L	32.3	36.3	15.5	10.7	10.0	5.3
M_p	32768	32768	32768	106 496	212 992	425 984

parameter $k_{\max}\eta$ (where $k_{\max} = \sqrt{2}N/3$ is the highest wavenumber resolved on an N^3 grid allowing for de-aliasing procedures necessary for pseudo-spectral methods) which is kept close to 1.5. This value is typical of much of the DNS literature in the degree to which the small scales are resolved, although it may not be sufficient [28] for high-order moments (of highly intermittent quantities) which however are not considered in this paper. Second is the overall simulation time T measured in units of T_L. Obviously, a large value of T/T_L is desirable but also expensive for large simulations, whereas the effects of a moderate T/T_L are mild for quantities which have time scales that are short compared with T_L (such as the dissipation rate at high Reynolds numbers). Finally, statistical errors are dependent on the ensemble size (M_p) of fluid particles tracked in the flow. An increase in M_p for the larger simulations is necessary so that the ensemble of particles can provide adequate sampling of the wider range of length scales present at higher Reynolds numbers.

3. Structure function and frequency spectrum

The second-order Lagrangian structure function has fundamental significance as the mean square of Lagrangian velocity increments $u^+(t+\tau) - u^+(t)$ which most stochastic models attempt to predict. (Note that here and elsewhere the superscript $^+$ denotes Lagrangian flow variables.) It is well understood that at small time lag $\tau \ll \tau_\eta$ (the Taylor-series limit of differentiability in time) $D_2^L(\tau) \approx \langle a^2\rangle\tau^2$, i.e. proportional to the acceleration variance, whereas at large $\tau \gg T_L$ (the diffusive limit of decorrelation at large time lags) $D_2^L(\tau) \approx 2\langle u^2\rangle$. As a result of these limiting behaviors the normalised structure function $D_2^L(\tau)/\langle\epsilon\rangle\tau$ (motivated by Equation 1) necessarily rises and falls like τ and $1/\tau$ at the limits of small and large τ respectively. This means a peak (C_0^*) or local maximum at intermediate time lags can always be expected, and is not by itself a sufficient indicator of a Lagrangian inertial range which requires a plateau of significant width.

Figure 1 shows the normalized structure function in Kolmogorov scaling, for the six simulations listed in table 1. It is clear that as the Reynolds number increases, these curves become more spread out (more so on the right) and increase in height, with a peak at several times that of τ_η. At the small τ limit the lack of a perfect "collapse" is consistent with at least a weak deviation from Kolmogorov scaling of the acceleration variance even at high Reynolds number, as discussed elsewhere (e.g. [11, 29]). Systematic growth of these curves toward large τ/τ_η is due to the increase with respect to τ_η of T_L which is the controlling time scale in the diffusive limit. In the inset it can be seen that although in the data range of our simulations C_0^* continues to increase with R_λ the increase is weaker at high R_λ. This suggests approach to an asymptotic constant is possible—incidentally at a value which is (within statistical error) quite consistent with recent estimates of 6–7 [5, 20] which are in turn in agreement with an earlier extrapolation based on stochastic modeling [15]. In addition, our 512^3 data at $R_\lambda \approx 240$ give $C_0^* \approx 5.0$ which is also quite comparable to 5.2 ± 0.8 at R_λ 284 quoted by Biferale et al. [30] also from DNS.

It should be noted that a substantial source of uncertainty in the task of inferring C_0 (or C_0^* as a function of Reynolds number) is in the value of $\langle\epsilon\rangle$. In experiments it is often impossible to measure $\langle\epsilon\rangle$ directly based on all components of velocity gradient fluctuations; in our DNS substantial variations in time are present [13] when averaged in space either from fixed grid points or instantaneous fluid particle positions. One way to check this is to divide the Lagrangian velocity time series into a number (say 8) of shorter segments and use the value of $\langle\epsilon\rangle$ averaged over such shorter intervals of time. Values of C_0^* obtained in this way are plotted in figure 2 versus the Reynolds number also averaged locally in time. The trend observed here

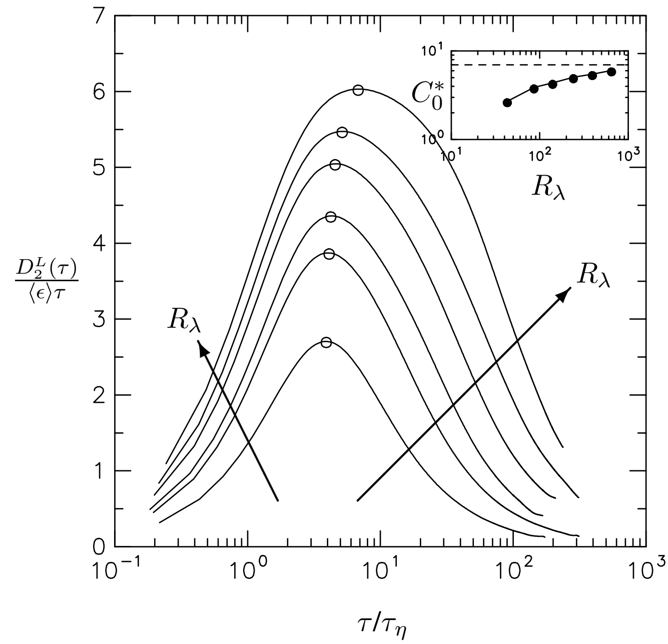

Figure 1. Second-order Lagrangian structure function scaled by Kolmogorov variables at six different Reynolds numbers for the simulations listed in table 1. Open circles mark the location of the peak in each curve. Arrows point in the direction of increasing Reynolds number. The inset shows these peak values (C_0^*) versus the time-averaged Reynolds number in each simulation, compared with a dashed line at the value 7.0.

is in broad agreement with the Reynolds number dependence inferred from figure 1. Within each cluster of points (for each simulation) larger values of C_0^* appear to be correlated with higher R_λ and lower $\langle\epsilon\rangle$: for this reason, the stronger trend observed in the 2048^3 simulation may be an artifact of limited simulation time (T) for averaging as noted in table 1. At the same

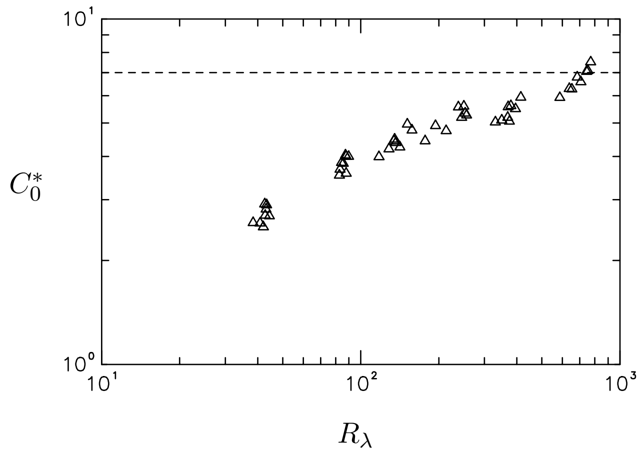

Figure 2. Scatter plot of C_0^* values versus Reynolds number obtained by dividing each simulation dataset into eight sub-periods and using the average $\langle\epsilon\rangle$ for each time period.

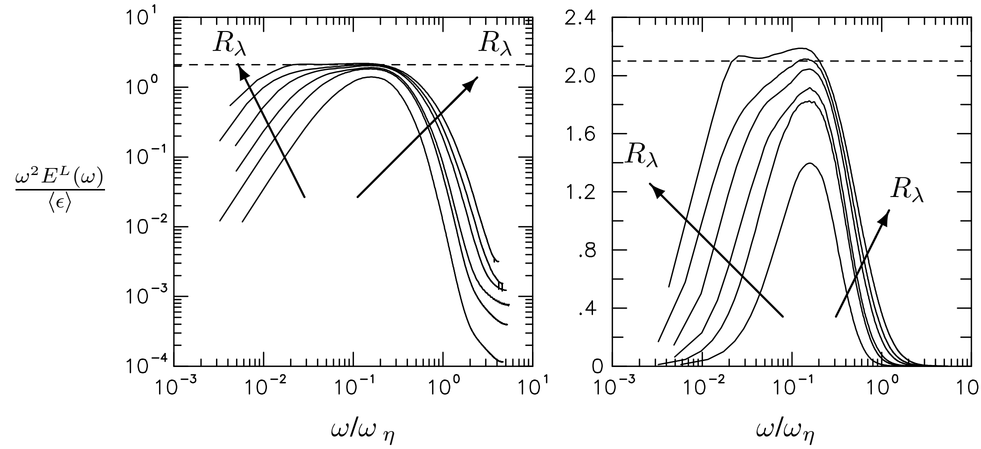

Figure 3. Lagrangian frequency spectrum of the velocity in Kolmogorov variables at different Reynolds numbers as in figure 1, using (a) log–log scales and (b) log–linear scales. Arrows point in the direction of increasing Reynolds number. The horizontal dashed lines are drawn at height 2.1.

time this also indicates the need for a future simulation at yet higher Reynolds number and grid resolution.

An alternative test of Lagrangian Kolmogorov similarity is through the velocity frequency spectrum, which is computed as the Fourier cosine transform of the velocity autocovariance $\langle u^+(t)u^+(t+\tau)\rangle$ (an even function of τ if the turbulence is statistically stationary). In the inertial range of frequencies $1/T_L \ll \omega \ll \omega_\eta$ (where $\omega_\eta = \pi/\tau_\eta$) the Kolmogorov similarity result is

$$E^L(\omega) = B_0\langle\epsilon\rangle\omega^{-2} \tag{2}$$

where the constant B_0 is equal to C_0/π [31]. In figure 3(a), 3(b) we show the normalized spectrum $\omega^2 E^L(\omega)/\langle\epsilon\rangle$ in log–log and log–linear scales respectively. It is clear that a significant scaling range exists for the 2048^3 data at highest Reynolds number, perhaps to a degree never observed before at least in DNS. Although there is no strong evidence that asymptotic values have been reached, the data suggest $B_0 \approx 2.1$ and hence $C_0 \approx 2.1\pi = 6.6$, which is about 10% higher than the peak of the curve at highest Reynolds number in figure 1. The 2048^3 curve is *very* flat even when viewed on a linear scale as in figure 3(b) where the values differ by less than 5% over one decade of frequencies, and the extent of this scaling is also similar to experimental data by Mordant et al. [32] quoted at R_λ 740. The increase in degree of flatness of this curve compared to the 1024 results seems very remarkable: this is not fully understood but may be related to the shift in peak locations seen in figure 1. A close comparison between figures 1 and 3(b) also shows that values of C_0 based on the frequency spectrum are slightly higher and less sensitive to Reynolds number than those obtained directly from the structure function. This difference in behavior is consistent with observations made by Lien and D'Asaro [16].

Although we present only second-order Lagrangian statistics in this paper it is clear that higher order Lagrangian structure functions can be used to study Lagrangian temporal intermittency, in the same way as Eulerian structure functions are commonly used to study the intermittency of spatial structure in turbulence. For example the classical Kolmogorov 1941 result for the nth-order Lagrangian structure function is a $(\langle\epsilon\rangle\tau)^{n/2}$ behavior in the inertial range, and anomalous scaling can be discussed by comparing actual scaling exponents versus $n/2$. We caution, however, that because Lagrangian quantities are more intermittent and require higher Reynolds number for similarity scaling, definitive conclusions free of ambiguity will be difficult to achieve.

4. Autocorrelations of dissipation quantities

An accurate knowledge of the Lagrangian properties of flow variables representing the small scales is important in current efforts in improving stochastic models via the incorporation of effects of intermittency: e.g. in [23], where a joint stochastic model of velocity, acceleration and the pseudo-dissipation is proposed. We consider here Lagrangian autocorrelations of the energy dissipation rate (ϵ), enstrophy (ζ) and pseudo-dissipation (φ), including in some cases their logarithms which are of interest in modeling approaches that invoke Kolmogorov's log-normal hypotheses [33]. Physically, these autocorrelation functions can provide qualitative information on, say, the typical time interval that a fluid particle may spend in a region of high strain rate (and/or vorticity), which is sensitive to the localized nature of such regions in space. Because these autocorrelations have approximately exponential forms, the simplest and most important measures of their correlation times are their integral time scales, which are obtained by numerical integration of a so-called unbiased estimate of the autocorrelation up to a sufficiently long time lag as described in [12]. In table 2 the integral time scales of these quantities are compared with the velocity integral time scale (T_L) and Kolmogorov time scale (τ_η).

Figures 4(a) and 4(b) show the autocorrelation of energy dissipation rate with time lag normalized by T_L and τ_η respectively. In figure 4(a) it is clear that the time scale of this autocorrelation decreases steadily relative to T_L as R_λ increases. This observation confirms a trend noted in [21] and supersedes previous results at low Reynolds number [13] where the range of time scales (between T_L and τ_η) was not sufficient to detect an unambiguous behavior.

In figure 4(b) we assess the degree to which the autocorrelation of the dissipation (as a feature of the small scales in turbulence) scales with the Kolmogorov time scale. The behavior at small τ, when the autocorrelation drops from 1 to 0.5 in about $2\tau_\eta$, is apparently universal under this scaling. For longer time lags the Reynolds number trend in figure 4(b) is clearly in reverse to that versus τ/T_L in figure 4(a). Correspondingly, as the Reynolds number increases the dissipation integral time scale (see Table 2) increases relative to τ_η, showing that behavior at larger time lags does not scale with τ_η. This suggests that the Lagrangian time history of dissipation is best considered as a process with two time scales, with T_L and τ_η each accounting for different aspects of the observed behavior. These observations are consistent with Pope [34] where it is suggested that the structure of dissipation (and related quantities) can be explained in terms of two time scales, with the relative contributions of each being

Table 2. Integral time scale information for energy dissipation (ϵ), enstrophy (ζ), pseudo-dissipation (φ) and their logarithms, normalized by Lagrangian velocity integral time scale (T_L) or Kolmogorov time scale (τ_η).

Grid	64^3	128^3	256^3	512^3	1024^3	2048^3
R_λ	43	86	140	240	393	648
$\mathcal{T}_\epsilon/T_L$	0.484	0.448	0.347	0.296	0.241	0.208
$\mathcal{T}_\epsilon/\tau_\eta$	2.60	3.85	4.55	5.86	7.49	9.11
$\mathcal{T}_{\ln\epsilon}/T_L$	0.544	0.534	0.464	0.453	0.420	0.441
$\mathcal{T}_{\ln\epsilon}/\tau_\eta$	2.93	4.68	6.08	8.97	13.0	19.3
$\mathcal{T}_\zeta/T_L$	0.974	0.773	0.542	0.370	0.247	0.181
$\mathcal{T}_\epsilon/\mathcal{T}_\zeta$	0.497	0.580	0.646	0.800	0.975	1.15
$\mathcal{T}_\varphi/T_L$	0.827	0.720	0.527	0.393	0.285	0.225
$\mathcal{T}_{\ln\varphi}/T_L$	0.915	0.829	0.679	0.599	0.516	0.512

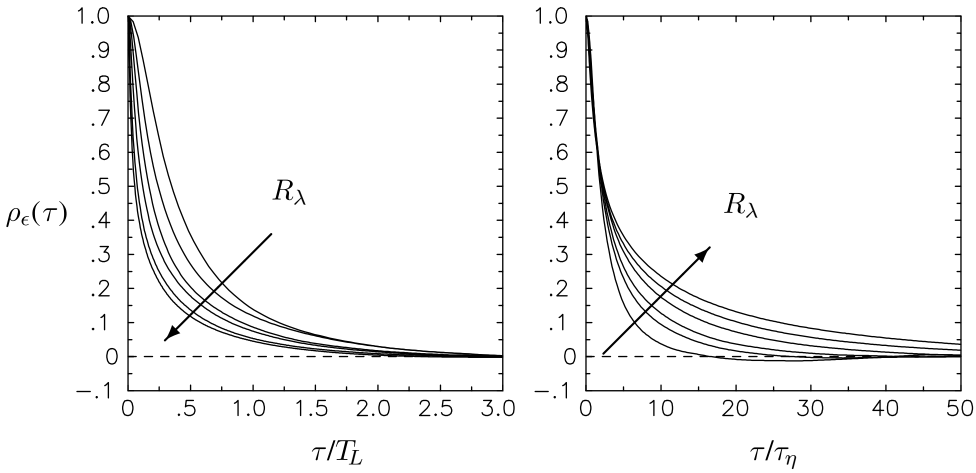

Figure 4. Lagrangian autocorrelation of dissipation, with time lag normalized by (a) the Lagrangian (velocity) integral time scale and (b) the Kolmogorov time scale. Arrows point in the direction of increasing Reynolds number.

dependent on the Reynolds number. As a purely empirical observation we also note that in all cases (from Table 2) $\mathcal{T}_\epsilon$ appears to be about 1.2–1.3 of $\sqrt{T_L \tau_\eta}$. Provided that the ratio T_L/τ_η is large then this result is qualitatively consistent with a multifractal theory prediction ([35]) that $\tau_\eta \ll \mathcal{T}_\epsilon \ll T_L$ at high Reynolds number.

In stochastic modeling it is useful to note that, despite some caveats associated with higher order moments [36], low-order statistics of the dissipation rate can be described reasonably well using a log-normal assumption. Table 2 includes data on the integral time scale of $\ln \epsilon$ which is found to follow T_L approximately while increasing strongly relative to τ_η at higher Reynolds number. There is considerable interest in modeling $\ln \epsilon^+(t)$ [22] or $\ln \varphi^+(t)$ [23, 37], as a first-order Markovian process characterized by exponential decay in the autocorrelation. To test this, we show in figures 5(a) and 5(b) the autocorrelations of $\ln \epsilon$ and $\ln \varphi$ with time lag τ normalized by the respective integral time scales.

It can be seen in figure 5(a) that the autocorrelation of $\ln \epsilon$ is closest to exponential for the lowest Reynolds number (line A) but deviates somewhat for all higher Reynolds numbers. This result suggests that Lagrangian modeling approaches which treat $\ln \epsilon^+(t)$ as a diffusion process with Gaussian statistics and known integral time scale (e.g. [22]) may be less accurate at high Reynolds number. In contrast deviations from exponential appear to be small for all Reynolds numbers in figure 5(b): that is, the Markovian modeling assumption has greater validity for $\ln \varphi^+(t)$. This observation helps explain improvements obtained in modeling [23] based on the logarithm of the pseudo-dissipation, which as the sum of all velocity gradients squared captures the effects of both local strain and rotation experienced by each fluid particle. A recent study of Eulerian statistics [11] has also shown that as a conditioning variable in the modeling of acceleration statistics the pseudo-dissipation has the advantage that it captures the intermittency of acceleration fluctuations most completely, such that the resulting conditional probability density (of acceleration given the pseudo-dissipation) is the easiest to describe.

In [11] some of the Eulerian properties of dissipation, enstrophy and pseudo-dissipation were studied over a Reynolds number range similar to the present data. Figures 6(a) and 6(b) compare the Lagrangian autocorrelations of these quantities including their respective logarithms at the lowest and highest Reynolds numbers available. In figure 6(a) (at $R_\lambda \approx 40$) the observations are similar to those found in previous work at lower Reynolds number by Yeung & Pope [13], namely that dissipation has a shorter correlation time than enstrophy

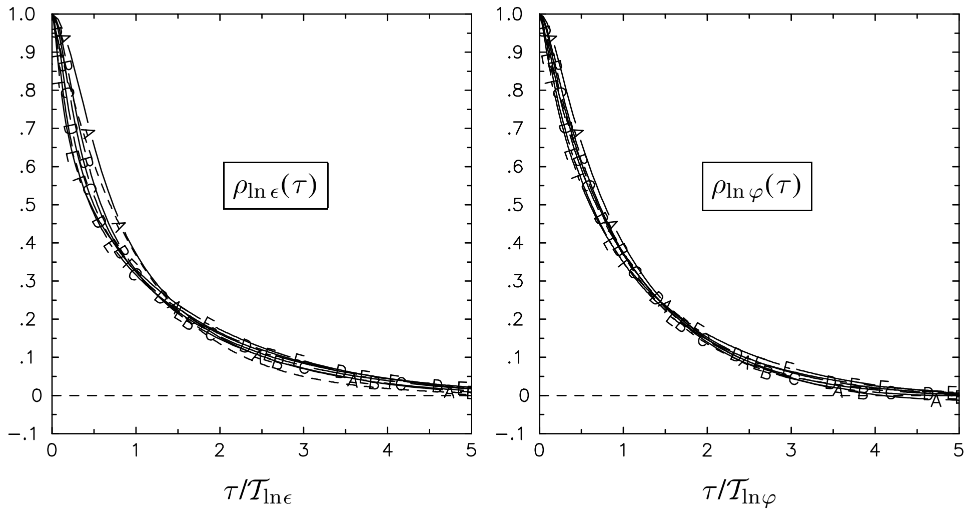

Figure 5. Lagrangian autocorrelation of (a) logarithm of dissipation and (b) logarithm of pseudo-dissipation, with time lag normalized by the integral time scale ($\mathcal{T}$) in each case. Lines A–F denote the six simulations listed in table 1 in order of increasing Reynolds number. A dashed curve (partly hidden) shows the exponential approximation $\exp(-\tau/\mathcal{T})$ for comparison.

and pseudo-dissipation, with enstrophy being correlated for longest, while the difference in behavior between each quantity and its logarithm is weak. In contrast figure 6(b) (at $R_\lambda \approx 650$) shows different trends at high Reynolds number: in this case ϵ, ζ and φ now have similar time scales, and de-correlate more rapidly than their logarithms. These observations are consistent with a general trend for dissipation and enstrophy to have similar statistics at high Reynolds number, while an increase in intermittency accounts for a greater difference between each

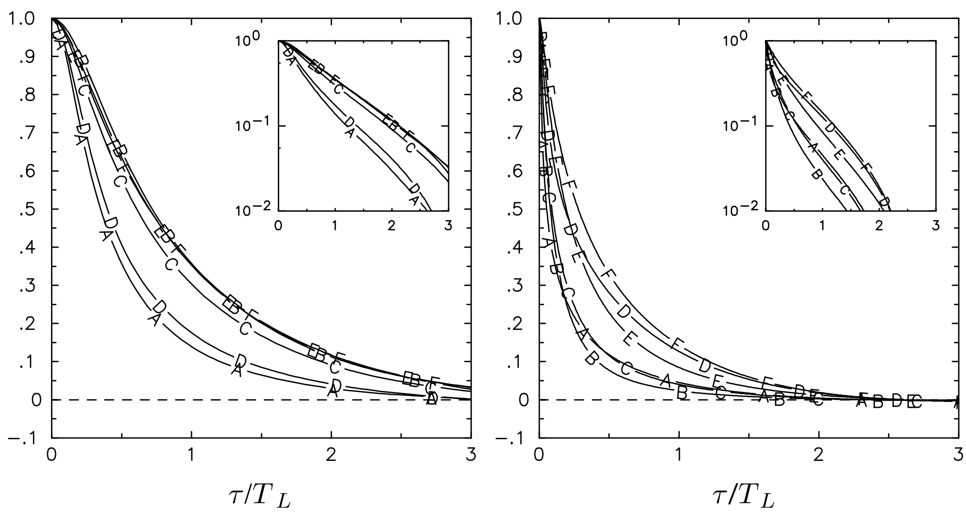

Figure 6. Comparison of the autocorrelations of the variables ϵ, ζ, φ (lines A–C) and their logarithms (D–F), at (a) the lowest and (b) highest Reynolds number in the present simulation database. Insets show the same data with the y-axis on a logarithmic scale and down to 0.01 (below which the autocorrelation values can be considered insignificant; exponential-decay behavior would be indicated by a straight line.

quantity and its logarithm. Finally, linear–log plots of the same data in the insets of this figure suggest approximate exponential decay (proportional to $\exp(-\tau)$) over a significant range of time lags where the autocorrelation decreases from close to 1.0 to less than 0.05.

5. Conclusions

In this paper we have provided a first report of Lagrangian statistics in stationary isotropic turbulence at the highest Reynolds number in the largest simulations to date using advanced Terascale supercomputer power provided at two national centers. We examine DNS data at grid resolutions from 64^3 to 2048^3 and at Taylor-scale Reynolds numbers from about 40 to 650. As in the past, we have tracked fluid particles using cubic-spline interpolation for the particle velocity and have saved time series of all the components of the velocity and velocity gradients—which also allow us to extract fluctuations of the dissipation, enstrophy and pseudo-dissipation representing local relative motion the flow. Tables 1 and 2 provide respectively information on the range of length and time scales, and on integral time scales of dissipation and related quantities.

The first objective in our data analysis has been to update previous results and compare with estimates of the Lagrangian Kolmogorov constant (C_0) in the second-order velocity structure function. Although the Reynolds number in the present data is still not sufficient to produce a fully unambiguous scaling range the results are, within a reasonable margin of error, nevertheless consistent with recent estimates of an asymptotic value in the range 6–7. We attempt to infer C_0 from the peaks of both the structure function and velocity frequency spectrum scaled by Kolmogorov variables, with the latter giving a slightly higher value. To address the effects of uncertainty caused by time-dependence of the space-averaged energy dissipation rate we divided data from each simulation into sub-intervals; the results are found to be self-consistent.

Our second objective is to study the Reynolds number dependence of autocorrelations and integral time scales of dissipation, enstrophy and pseudo-dissipation. As the Reynolds number increases, the Lagrangian time scale of dissipation decreases relative to the velocity integral time scale but increases relative to the Kolmogorov time scale, thus suggesting modeling as a stochastic process with two time scales. The logarithm of the pseudo-dissipation has an autocorrelation which is close to exponential and provides support for modeling as a diffusion process. In contrast to previous results at low Reynolds number, at the highest Reynolds number in this paper we find that the autocorrelations of dissipation, enstrophy and pseudo-dissipation are close together whereas the autocorrelations of their logarithms have significantly longer time scales. These Reynolds number effects are consistent with other observations that the statistics of dissipation and enstrophy become progressively closer to each other. The results presented in this paper can be combined with our recent work on Eulerian conditional acceleration statistics to help develop a new stochastic model that can account for intermittency successfully through the pseudo-dissipation.

Acknowledgements

We gratefully acknowledge support from the National Science Foundation, via Grant CTS-0328314 (PKY) and CTS-0328329 (SBP), and via advanced resources at Pittsburgh Supercomputing Center (PSC) and San Diego Supercomputer Center (SDSC). The 2048^3 simulation reported in this work was conducted on the PSC's 3000-processor Compaq Alpha Cluster, with extraordinarily dedicated assistance from our strategic consultant, David O'Neal. At Georgia Tech, Diego Donzis and Elizabeth Kurth also contributed to the conduct of these simulations. Finally, PKY wishes to thank Professor K.R. Sreenivasan of ICTP, Italy for facilitating his participation at the Challenging Turbulent Lagrangian Dynamics Workshop, Roma, Italy (September 2005).

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