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FLAMELET AND DISTRIBUTED COMBUSTION IN PREMIXED TURBULENT FLAMES

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Calculations are reported that compare the properties of idealized premixed flames in which either flamelet or distributed combustion occurs. The calculations are based on the Monte Carlo solution of a modelled transport equation for the joint probability density function (pdf) of the velocities and the reaction progress variable. In the joint pdf equation, it is the molecular diffusion term that is fundamentally different for the two types of combustion, and consequently different models for the term are employed. For flamelet combustion, at moderate and high Damkohler numbers, the progress variable pdf is a double-delta-function distribution, and the turbulent flame speed is more than twice the turbulence intensity (independent of the Damkohler number). For distributed combustion, on the other hand, even at high Damkohler number, the progress variable pdf shows significant probability of partially reacted fluid. The flame speed is less than in flamelet combustion and increases with Damkohler number.

Introduction

Premixed turbulent flames are important because of their occurrence in spark ignition engines[1,2] and because they are fundamental to our understanding of yet more complicated combustion phenomena.[3] Their behavior is difficult to describe theoretically because several interconnected processes—reaction, diffusion, volume expansion—occur in an inhomogeneous stochastic flow field.[4] The turbulence can have a dominant influence on reaction and diffusion, while the volume expansion can have a significant effect on the turbulence.

The present work is concerned with the influence of the micro-scale structure of the flame on macro-scale properties—the turbulent flame speed, for example. Two extreme cases are studied: *flamelet combustion*, in which combustion occurs in thin sheets which, locally, have the properties of undisturbed laminar flames; and *distributed combustion*, in which reaction is distributed more uniformly in space and is not necessarily accompanied by steep spatial concentration gradients. Flamelet combustion occurs when the laminar flame thickness δ_ℓ is much smaller than the Kolmogorov scale η whereas, conversely, distributed combustion occurs for $\delta_\ell/\eta >> 1$.

Flamelet and distributed combustion are studied in an idealized premixed turbulent flame in which many of the previously-mentioned complicating difficulties are absent. The flame is statistically stationary and one-dimensional, and propagates through high-Reynolds-number non-decaying homogeneous turbulence. The density and transport properties are constant, and a one-step reaction is assumed so that the instantaneous thermochemical state is uniquely related to a progress variable $\phi(x,t)$. (The progress variable $\phi(x,t)$—denoted by c in the Bray-Moss model[5]—is zero in the unburnt reactants and unity in the burnt products.) The consideration of this idealized case has the advantage that constant-density turbulence-modelling assumptions can be used with confidence. On the other hand it has the significant disadvantage of being too far removed from real flames to allow a direct comparison between theory and experiment.

The theoretical approach adopted is to solve a modelled transport equation for the joint probability density function (pdf) of the velocities $u(x,t)$ and the progress variable $\phi(x,t)$.[6] In this approach, reaction and convection (by mean and fluctuating velocities) are treated exactly, while the effects of pressure fluctuations and molecular transport have to be modelled. We use models for the effects of pressure fluctuations and molecular viscosity that have been developed and tested for turbulent shear flows.[7,8] In flamelet and distributed combustion, the micro-scale structure of $\phi(x,t)$ is quite different and hence different models for the effects of molecular diffusion are needed. These are described in the next section.

The sole dimensionless parameter is the

Damkohler number Da, which is the turbulence-to-reaction time scale ratio. For both flamelet and distributed combustion, the modelled joint pdf equation was solved by a Monte Carlo method[7,9] for a wide range of Damkohler numbers. The results reported in the third section include the turbulent flame speed S_T, profiles of the mean and variance of ϕ, as well as the progress variable pdf.

Theory

The idealized flame considered propagates through constant (unit) density, homogeneous, isotropic, non-decaying turbulence. The velocity $u(x,t)$, which has zero mean ($\langle u \rangle = 0$), satisfies the equations

$$\frac{\partial u_i}{\partial x_i} = 0, \tag{1}$$

and

$$\frac{Du_i}{Dt} = \nu \nabla^2 u_i - \frac{\partial p}{\partial x_i} + \frac{1}{2}\frac{\epsilon}{k} u_i, \tag{2}$$

where $p(x,t)$ is the pressure and ν is the viscosity. The last term in Eq. (2) represents a source of turbulence energy that exactly balances the rate of viscous dissipation ϵ. Thus the turbulent kinetic energy $k \equiv 1/2\langle u_i u_i \rangle$ remains constant and uniform.

The progress variable $\phi(x,t)$ evolves according to the equation

$$\frac{D\phi}{Dt} = \nu \nabla^2 \phi + S(\phi). \tag{3}$$

The use of ν as the transport coefficient implicitly assumes unit Lewis and Prandtl numbers. The source $S(\phi)$ represents the rate of change of ϕ due to reaction and is taken to be

$$S(\phi) = S^*(\phi)/\tau_R,$$

where τ_R is the reaction time scale, and the normalized rate $S^*(\phi)$ is given by the Arrhenius expression

$$S^*(\phi) = 6.11 \times 10^7 \phi(1-\phi) \cdot \exp\{-30{,}000/(300 + 1{,}800\phi)\}. \tag{4}$$

This expression, which is plotted on Fig. 1, corresponds to an activation temperature of 30,000 K and unburnt and burnt temperatures of 300 and 2,100 K. The numerical constant is chosen so that the maximum value of $S^*(\phi)$ is unity.

For reaction the characteristic time scale is τ_R. For the turbulence the characteristic time scale is $\tau \equiv k/\epsilon$, the velocity scale is $u' \equiv (2k/3)^{1/2}$, and the length scale is $\ell \equiv u'\tau$. From these scales and from the viscosity ν, two dimensionless parameters can be formed. We take these to be the Reynolds number $\mathrm{Re} \equiv u'\ell/\nu$ and the Damkohler number $\mathrm{Da} \equiv \tau/\tau_R$. Any other dimensionless group—S_ℓ/u', δ_ℓ/ℓ or δ_ℓ/η, for example—is uniquely related to Re and Da. The Reynolds number is assumed to be very large, but—according to the modelled equations—its value does not affect the results (except to determine the type of combustion). The value of the Damkohler number, on the other hand, is of prime importance.

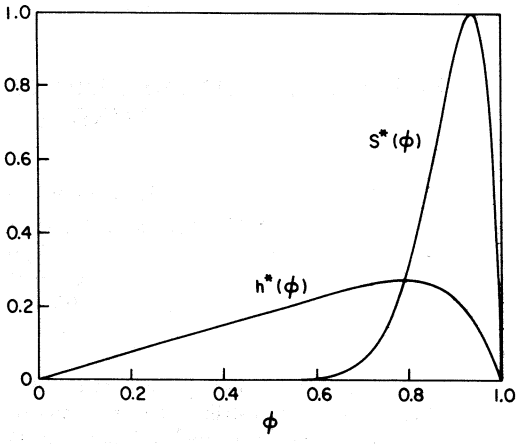

FIG. 1. Normalized reaction rate and laminar flame function against progress variable.

As discussed by Bray,[4] McNutt,[13] and Stambuleanu,[15] different regimes of premixed turbulent combustion occur for different values of the two dimensionless groups. Distributed combustion occurs for $\mathrm{Re} >> 1$ and $\mathrm{Da} << \mathrm{Re}^{1/2}$, from which it follows[13] $\delta_\ell/\eta >> 1$ and $S_\ell/u' << 1$, (where S_ℓ is the laminar flame speed). Flamelet combustion occurs for $\mathrm{Re} >> 1$ and $\mathrm{Re}^{1/2} << \mathrm{Da} << \mathrm{Re}$, from which it follows[13] $\delta_\ell/\eta << 1$ and $S_\ell/u' << 1$.

The flame is statistically one-dimensional, with statistical quantities varying only in the x_1 direction. At $x_1 = -\infty$ there are pure products ($\phi(-\infty) = 1$) while at $x_1 = \infty$ there are pure reactants ($\phi(\infty) = 0$). The flame propagates at the turbulent flame speed S_T in the x_1 direction and is statistically stationary (in a coordinate system moving at the speed S_T). It is emphasized that the flame speed S_T is determined as part of the solution.

The joint pdf $f(V,\psi;x,t)$ is defined to be the probability density of the simultaneous events $u(x,t) = V$ and $\phi(x,t) = \psi$. As written, the joint pdf $f(V,\psi;x,t)$ is a function of eight independent variables—three velocity variables V_1,V_2,V_3, the composition variable ψ, three spatial variables and time. But because the flame is statistically one-dimen-

sional and stationary, three independent variables can be eliminated. For example, defining

$$f_o(V,\psi;x_1) \equiv f(V,\psi;x_1,0,0,0), \tag{5}$$

we have, at any (x,t),

$$f(V,\psi;x,t) = f_o(V,\psi;x_1 - S_T t). \tag{6}$$

The boundary conditions on f are

$$f(V,\psi;-\infty,x_2,x_3,t) = g(V)\delta(1-\psi), \tag{7}$$

and

$$f(V,\psi;\infty,x_2,x_3,t) = g(V)\delta(\psi), \tag{8}$$

where $g(V)$ is the joint pdf of the velocities.

An exact evolution equation for $f(V,\psi;x,t)$ can be derived[6,9] from Eqs. (1)–(3):

$$\frac{\partial f}{\partial t} + V_i\frac{\partial f}{\partial x_i} + \frac{\partial}{\partial \psi}(fS(\psi)) + \frac{1}{2\tau}\frac{\partial}{\partial V_i}(fV_i) = -\frac{\partial}{\partial V_i}\left\{f\left\langle \nu\nabla^2 u_i - \frac{\partial p}{\partial x_i}\middle| V,\psi\right\rangle\right\} - \frac{\partial}{\partial \psi}\{f\langle \nu\nabla^2\phi|V,\psi\rangle\}. \tag{9}$$

The terms on the left-hand side are exact and represent, respectively the change with time, convection, reaction, and the source of turbulence energy. The terms on the right-hand side of the equation contain (as unknowns to be modelled) conditional expectations. (For any quantity Q, $\langle Q|V,\psi\rangle$ denotes the expectation conditional upon the events $u(x,t) = V$, $\phi(x,t) = \psi$.)

As in previous work,[6,7,10] the effect of the fluctuating pressure gradient is modelled by the stochastic reorientation model; and the effect of dissipation is modelled by the improved stochastic mixing model.[7,8] The corresponding modelled equation for the Reynolds stresses $\langle u_i u_j\rangle$ is

$$\frac{d}{dt}\langle u_i u_j\rangle - \langle u_i u_j\rangle/\tau = -C_R(\langle u_i u_j\rangle - 2/3\, k\,\delta_{ij})/\tau - 2/3\,\epsilon\delta_{ij}. \tag{10}$$

It may be seen that this corresponds to the assumption of isotropic dissipation and Rotta's[11] return to isotropy model. Following Launder, Reece and Rodi,[12] we choose the Rotta constant to be $C_R = 1.5$.

We now consider the modelling of the crucial molecular diffusion term—first for *distributed combustion*. In this case the Kolmogorov scale η is much smaller than the laminar flame thickness δ_ℓ. It follows from scaling arguments[13] that the Kolmogorov time scale τ_η is much smaller than the reaction time scale τ_R or, equivalently, than the laminar flame transit time $\tau_\ell \equiv \delta_\ell/S_\ell$ (where S_ℓ is the laminar flame speed). Consequently the steepest gradients of ϕ—which provide the dominant contribution to $\langle \nu\nabla^2\phi|V,\psi\rangle$—are due to turbulent straining, and are not associated with reaction fronts. Thus the microscale is oblivious to the presence of reaction, and the molecular diffusion term can be modelled in the same way as for inert flows: we again use the improved stochastic mixing model.[7,8]

With this modelling of the molecular diffusion term appropriate to distributed combustion, the corresponding model equations for the variance $\langle \phi'^2\rangle$ and the flux $\langle u_i\phi'\rangle$ are

$$\frac{\partial}{\partial t}\langle\phi'^2\rangle + \frac{\partial}{\partial x_i}\langle u_i\phi'^2\rangle + 2\langle u_i\phi'\rangle\frac{\partial\langle\phi\rangle}{\partial x_i} - 2\langle\phi' S(\phi)\rangle = -C_\phi\langle\phi'^2\rangle/\tau, \tag{11}$$

and

$$\frac{\partial}{\partial t}\langle u_i\phi'\rangle + \frac{\partial}{\partial x_j}\langle u_i u_j\phi'\rangle + \langle u_i u_j\rangle\frac{\partial\langle\phi\rangle}{\partial x_j} - \frac{\langle u_i\phi'\rangle}{2\tau} - \langle u_i S(\phi)\rangle = -C_M\langle u_i\phi'\rangle/\tau. \tag{12}$$

The terms on the left-hand side are exact, while those on the right-hand side are modelled. The model constants are $C_\phi = 2.0$ and $C_M = 3.1$. (It is emphasized that these moment equations, Eqs. (10)–(12), are presented solely to demonstrate the effect of the models on the second moments. The equation solved is for the joint pdf f.)

For *flamelet combustion* the picture is quite different. The Kolmogorov scale η is much larger than the laminar flame thickness δ_ℓ and, it follows,[13] the time scale τ_η is much larger than the reaction time scale τ_R (or the laminar flame transit time τ_ℓ). Consequently a laminar flamelet suffers little turbulent straining and its local radius of curvature (of order η) is large compared to δ_ℓ. Thus, locally, the flamelet behaves like an unstrained plane laminar flame. (It should be noted that this conclusion follows from the assumed strong inequality $Da >> Re^{1/2}$. In practice, this strong inequality may not hold, and the straining of laminar flamelets may be an important phenomenon.[4])

The evolution equation for $\phi(x,t)$, Eq. (3), admits a solution corresponding to a plane laminar flame propagating in the x_1 direction. From this solution, quantities such as

$$H \equiv \nu\nabla^2\phi + S(\phi), \tag{13}$$

can be determined as functions of x_1. But since the

solution $\phi(x_1)$ is a monotonic function of x_1, H can be expressed as a function of ϕ. Thus, there is a function $h(\psi)$ such that in a plane laminar flame

$$h(\psi) = [\mathrm{H}]_{\phi=\psi} = [\nu\nabla^2\phi + \mathrm{S}(\phi)]_{\phi=\psi}. \quad (14)$$

Equation (3) was solved numerically to determine the laminar flame profile and hence $h(\psi)$. The normalized function $h^*(\psi) = h(\psi)\tau_R$ is shown on Fig. 1.

The consequence of these observations for the joint pdf equation is remarkable. The reaction and diffusion terms in Eq. (9) can be combined to yield

$$\frac{\partial f}{\partial t} \ldots = -\frac{\partial}{\partial \psi}\{f\langle \nu\nabla^2\phi + \mathrm{S}(\phi)|V,\psi\rangle\}$$

$$= -\frac{\partial}{\partial \psi}\{f\langle \mathrm{H}|V,\psi\rangle\}, \quad (15)$$

(where the dots indicate the remaining terms in Eq. (9)). Now for flamelet combustion, it is assumed that locally the flamelet behaves like a plane laminar flame, in which case H is a unique function of ϕ. Thus

$$\langle \mathrm{H}|V,\psi\rangle = h(\psi), \quad (16)$$

and the effect of reaction and diffusion on the joint pdf evolution is, simply,

$$\frac{\partial f}{\partial t} \ldots = -\frac{\partial}{\partial \psi}\{fh(\psi)\}. \quad (17)$$

Equation (17) shows that, under the extreme assumption of flamelet combustion, reaction and diffusion can be expressed in closed from in the joint pdf equation in terms of the known function $h(\psi)$. Thus, in contrast to the situation in inert flows and distributed combustion, the diffusion term does not have to be modelled. This conclusion requires qualification. It has been assumed that the gradients of ϕ in the flamelet are of order $1/\delta_\ell$ and hence are much greater than those caused by turbulent straining (of order $1/\eta$). This assumption is justified in the central part of the flame, but not remote from the flame where ϕ approaches zero or unity. Indeed, the laminar flame solution shows that all spatial derivatives of ϕ tend to zero as ϕ tends to zero or unity. In these outer regions, turbulent straining causes the steepest gradients of ϕ and it is appropriate, therefore, to model the diffusion term in the same way as for inert flows or for distributed combustion.

The approach we adopt is to use Eq. (17) *and also* the improved mixing model. Close to the extreme values of ϕ (zero and one) the diffusion term is modelled appropriately by the mixing model and the contribution from $h(\psi)$ is negligible. Conversely, for intermediate values of ϕ, the term in $h(\psi)$ correctly accounts for the effect of diffusion, and the contribution from the mixing model is negligible in comparison. It is interesting to note that the resulting modelled joint pdf equation for flamelet combustion is identical to that for distributed combustion except that $h(\psi)$ replaces $\mathrm{S}(\psi)$.

Starting from an almost arbitrary initial condition, the modelled joint pdf equation is solved by a Monte Carlo method.[9,10] The results reported in the next section were obtained after the statistically-stationary state had been reached.

Results and Discussion

For one condition (distributed combustion, Da = 100) the profiles of the mean and variance of the progress variable ϕ are shown on Fig. 2. The spatial coordinate x_1 is normalized by the turbulent length scale $\ell \equiv u'\tau$, and the choice of origin is immaterial. The flame propagates to the right into the pure reactants ($\phi = 0$), leaving products ($\phi = 1$) behind it. The variance $\langle\phi'^2\rangle$ is zero outside the flame and reaches a peak close to where $\langle\phi\rangle = 1/2$.

It may be seen that both $\langle\phi\rangle$ and $\langle\phi'^2\rangle$ go rapidly to their boundary values on the product side ($x_1 < 0$), but they do so slowly on the reactant side ($x_1 > 0$). On the product side reaction is the dominant process. Fluid with intermediate values of ϕ rapidly react ($\tau_R = \tau/100$) to attain the value $\phi = 1$. On the reactant side, where ϕ is small, so also is the reaction rate (see Fig. 1). Thus fluid attains higher values of ϕ by the relatively slow ($\tau = 100\tau_R$) mixing process.

On Fig. 3, the turbulent flame speed S_T normalized by the turbulence intensity u' is shown as a function of Damkohler number for both kinds of combustion. For flamelet combustion, an asymptotic value of $S_T/u' = 2.1$ is attained for Da > 5. This flame speed, it should be recognized, is very high. The only transport mechanism in physical space is convection, and yet the probability of fluid travelling at a speed greater than $2.1\ u'$ is less than 2%.[1] For distributed combustion, on the other hand,

[1] A reviewer of this paper observed that the calculated flame speed $S_T \approx 2u'$ is in accord with some empirical correlations.[16-18] While this agreement is encouraging, there are three reasons why we are reluctant to draw firm conclusions from the comparison: first, the idealized flame considered here differs in several important respects from real flames; second, it is not clear that the experimental configurations conform very closely to a statistically-stationary one-dimensional normal flame; and third, there are numerous empirical correlations,[16] only some of which yield (in the appropriate limit) $S_T \approx 2u'$.

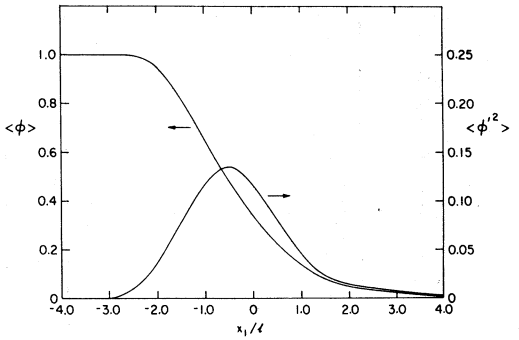

FIG. 2. Mean and variance of progress variable against normalized distance. Distributed combustion, Da = 100.

significantly different behavior is observed. The flame speed is lower, and by Da = 10^4 it is still increasing linearly with $\ell n(Da)$. The reasons for these differences are discussed below. (For flamelet combustion, although results are reported for Da = 1, it should be noted that the condition $\delta_\ell/\eta << 1$ implies Da >> $Re^{1/2}$. Thus, at moderate or high Reynolds number, flamelet combustion occurs only at high Damkohler number.)

The flame thickness δ normalized by the turbulent length scale is shown on Fig. 4. The thickness can be defined in many ways: for reasons of computational stability, we choose to define δ to be the standard deviation of the distribution

$$q(x_1) \equiv \langle\phi\rangle(1 - \langle\phi\rangle) \Big/ \int_{-\infty}^{\infty} \langle\phi\rangle(1 - \langle\phi\rangle)dx_1. \quad (18)$$

It may be seen from the figure that, for both types

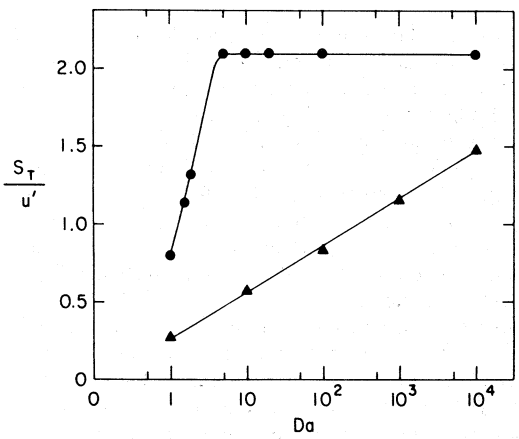

FIG. 3. Normalized turbulent flame speed against Damkohler number
● Flamelet combustion
▲ Distributed combustion

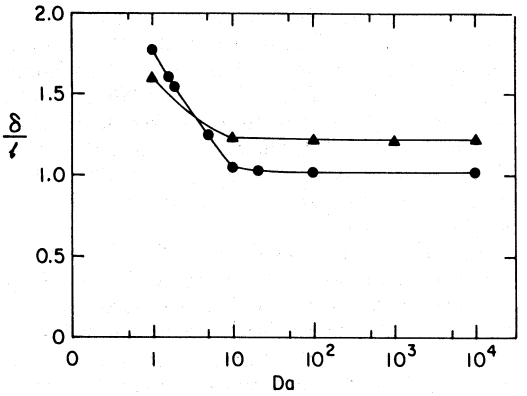

FIG. 4. Normalized turbulent flame thickness against Damkohler number.
● Flamelet combustion
▲ Distributed combustion

of combustion, the flame thickness decreases with increasing Da, reaching asymptotic values of $\delta/\ell \approx 1$ by Da = 10.

We now examine the variance $\langle\phi'^2\rangle$. Since $\langle\phi\rangle$ is a monotonic function of x_1 (see Fig. 2), $\langle\phi'^2\rangle$ can be plotted as a function of $\langle\phi\rangle$. This is done for flamelet and distributed combustion on Figs. 5 and 6, respectively. For a given value of $\langle\phi\rangle$, the variance reaches its maximum possible value of $\langle\phi\rangle$ $(1 - \langle\phi\rangle)$ when the pdf of ϕ, $f_\phi(\psi)$, is com-

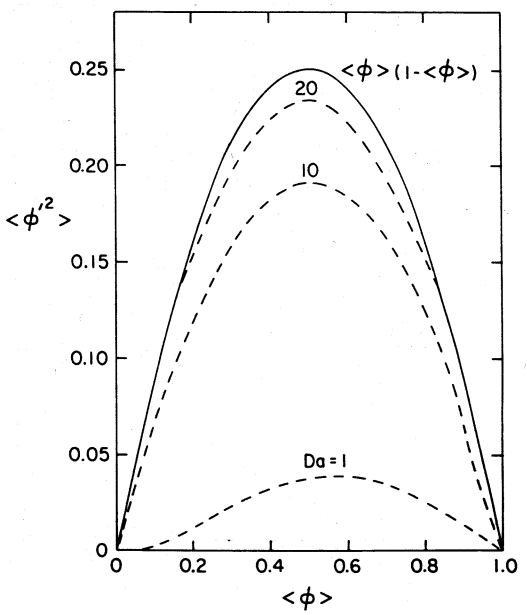

FIG. 5. Progress variable variance against mean for different Damkohler numbers. Flamelet combustion.

posed of delta functions at $\psi = 0$ and $\psi = 1$. It may be seen from Fig. 5 that for flamelet combustion the variance increases with Damkohler number and that for Da = 20 it is close to its extreme value and hence $f_\phi(\psi)$ is close to a double-delta-function distribution. For distributed combustion (Fig. 6) a slower increase of $\langle\phi'^2\rangle$ with Da is evident: even for Da = 10^4 the variance is significantly less than its extreme value.

The different flame behavior for flamelet and distributed combustion is due to the different shapes of $S^*(\psi)$ and $h^*(\psi)$, Fig. 1. For both types of combustion, at high Damkohler number, the rate-controlling process is the turbulent mixing of pure reactants ($\phi = 0$). For flamelet combustion, mixing has to increase ϕ from zero only a small amount before the rate of increase of ϕ (i.e. $h(\phi)$) becomes large. With Da = 10^4, for example, at $\phi = 0.01$ we have $h(\phi) \approx 40/\tau$. Once the fluid attains a value of ϕ greater than zero (0.01, say) it rapidly reacts to a value close to 1 (0.99 say). Consequently, the probability of ϕ having an intermediate value $0.01 < \phi < 0.99$ is very small, and, to a good approximation, the pdf $f_\phi(\psi)$ is a double-delta-function distribution.

In distributed combustion, turbulent mixing has to increase ϕ significantly before the rate of increase of ϕ due to reaction (i.e. $S(\phi)$) becomes large. With Da = 10^4, at $\phi = 0.4$, the reaction rate is small: $S(\phi) = 0.025/\tau$. As the Damkohler number increases, the value to which ϕ has to be raised

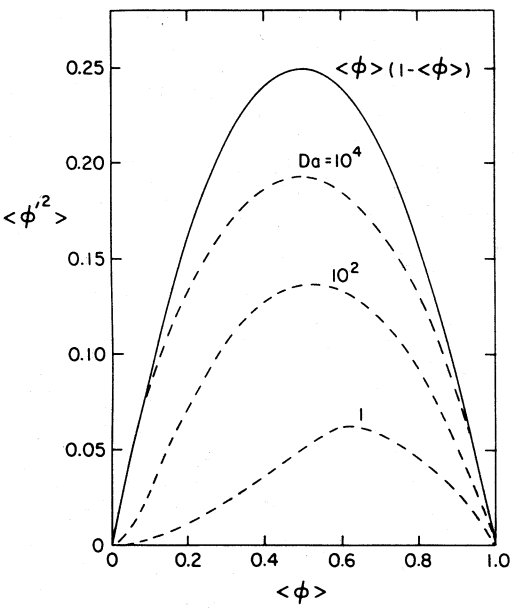

FIG. 6. Progress variable variance against mean for different Damkohler numbers. Distributed combustion.

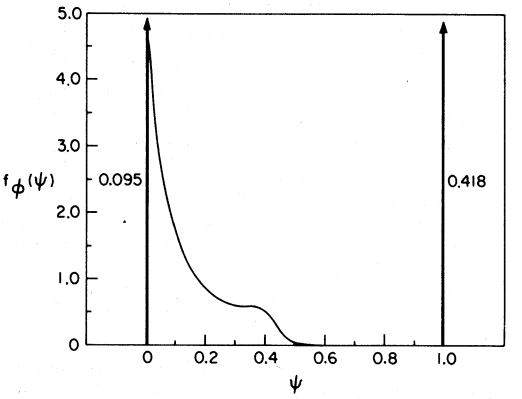

FIG. 7. Progress variable pdf at the location where $\langle\phi\rangle = 1/2$. Distributed combustion, Da = 10^4.

by mixing decreases—but extremely slowly. With Da = 10^7, at $\phi = 0.3$, the reaction rate is just $0.04/\tau$. It is for this reason that the flame speed rises slowly with Damkohler number, Fig. 3.

A pdf of the progress variable in distributed combustion is shown on Fig. 7. The Damkohler number is 10^4 and the pdf $f_\phi(\psi)$ is reported at the location where $\langle\phi\rangle = 1/2$. There are spikes (with probabilities 0.095 and 0.418) at zero and unity corresponding to pure reactants and products. But it is the shape of the intermediate distribution that confirms the validity of the reasoning given above. In the range $0.55 < \psi < 0.99$ the reaction rate is greater than $12/\tau$ and the probability is negligible: in the range $0 < \psi < 0.45$ the reaction is less than $0.3/\tau$ and there is significant probability.

Conclusion

A modelled transport equation for the joint pdf of velocity and the progress variable has been solved by a Monte Carlo method for idealized premixed turbulent flames. Different models have been used for distributed and flamelet combustion.

Significant differences are found between the two types of combustion. In flamelet combustion, the progress variable pdf adopts a double-delta-function distribution for moderate and high Damkohler numbers (Da > 20). For Da ≥ 5 the turbulent flame speed has the high value $S_T = 2.1\ u'$. For distributed combustion, on the other hand, even at high Damkohler number the progress variable pdf shows significant probability over a range of low ϕ values. As the Damkohler number increases, this range decreases slightly and the flame speed increases slightly. At Da = 10^4 the flame speed is $S_T \approx 1.5\ u'$ and appears to increase linearly with $\ell n(Da)$.

These results support the assumption of a double-delta-function distribution for flamelet combus-

tion that is a corner stone of the Bray-Moss model.[4,5]

The eddy-break-up model for distributed combustion[14] assumes that (at high Damkohler number) turbulent mixing is the rate-controlling process and so the mean reaction rate (and hence the flame speed) scale with turbulence quantities, independent of Da. In the present model it is also found that turbulent mixing is the rate-controlling process. But the amount of mixing required before reaction becomes rapid depends on Da. Consequently the flame speed also depends on the Damkohler number.

Nomenclature

C_M, C_R, C_ϕ	turbulence model constants
Da	Damkohler number (τ/τ_R)
$f(V,\psi;x,t)$	joint pdf of $u(x,t)$ and $\phi(x,t)$
$f_\phi(\psi;x,t)$	pdf of $\phi(x,t)$
$g(V)$	joint pdf of $u(x,t)$
$h(\psi)$	laminar flame function (Eq. 14)
$h^*(\psi)$	normalized laminar flame function ($\tau_R h(\psi)$)
k	turbulent kinetic energy ($3/2\ u'^2$)
ℓ	turbulent length scale ($u'\tau$)
$p(x,t)$	pressure
Re	Reynolds number ($u'\ell/\nu$)
$S(\phi), S^*(\phi)$	reaction rate, normalized reaction rate
S_ℓ, S_T	laminar and turbulent flame speeds
t	time
$u(x,t)$	velocity
u'	turbulence intensity
V	independent velocity variables
x	position

Greek Letters

δ_ℓ, δ	laminar and turbulent flame thicknesses
ϵ	rate of dissipation of turbulent kinetic energy
η	Kolmogorov length scale
ν	viscosity
$\tau, \tau_\ell, \tau_R, \tau_\eta$	time scales: turbulent; laminar flame (δ_ℓ/S_ℓ); reaction; Kolmogorov
$\phi(x,t)$	reaction progress variable
$\phi'(x,t)$	fluctuation in ϕ ($\phi - \langle\phi\rangle$)
ψ	independent reaction progress variable

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REFERENCES

1. Heywood, J. B.: Prog. Energy Combust. Sci. *1*, 135 (1976).
2. Keck, J. C.: *Nineteenth Symposium (International) on Combustion*, p. 1451, The Combustion Institute, 1983.
3. Strahle, W. C.: *Nineteenth Symposium (International) on Combustion*, The Combustion Institute, 1983.
4. Bray, K. N. C.: in *Turbulent Reactive Flows*, ed. Libby, P. A. and Williams, F. A., Springer-Verlag, 1980.
5. Bray, K. N. C. and Moss, J. B.: University of Southampton Report AASU 335, 1974.
6. Pope, S. B.: Phys. Fluids *24*, 588 (1981).
7. Pope, S. B: *AIAA J 22*, 896 (1984).
8. Pope, S. B.: Combust. Sci. Technol. *28*, 131 (1982).
9. Pope, S. B.: *PDF Methods in Turbulent Reactive Flows*, to be published in Prog. Energy Combust. Sci. (1984).
10. Pope, S. B.: *in Turbulent Shear Flows 3*, ed. Bradbury L. J. S., et al., Springer-Verlag, 1982.
11. Rotta, J. C.: Z. Phys. *129*, 547 (1951).
12. Launder, B. E., Reece, G. J. and Rodi, W.: J. Fluid Mech. *68*, 537 (1975).
13. McNutt, D. G.: M.S. Thesis, M.I.T., 1981.
14. Mason, H. B. and Spalding, D. B.: Comb. Inst. European Symp. (ed. F. J. Weinberg) p. 601, Academic Press, 1973.
15. Stambuleanu, A.: *Flame Combustion Processes in Industry*, Abacus, Tunbridge Wells, England, 1976.
16. Andrews, G. E., Bradley, D. and Lwakabamba, S. B.: Combust. Flame *24*, 285 (1975).
17. Wohl, K., Shore, L., Rosenberg, H. and Weil, C. W.: *Fourth Symposium (International) on Combustion*, p. 620, Williams and Wilkins, Baltimore, 1953.
18. Wohl, K.: Ind. Eng. Chem. *47*, 825, 1955.

COMMENTS

R. W. Bilger, University of Sydney. What are the implications of this work for finite rate kinetic effects in turbulent non-premixed flames? Will the molecular mixing term in the joint PDF equation need to include chemistry effects in its modelling? What sort of errors can be expected if the normal coalescence/dispersion modelling as for conserved scalars is used?

Authors' Reply. In order to account for finite-rate kinetic effects in turbulent diffusion flames, we need to consider the joint pdf of the mixture fraction (f) and of a perturbation variable (z) that measures departures from equilibrium. The transport equation for z contains a source term, the characteristic time scale of which is τ_z. According to our modelling, if τ_z is much greater than the Kolmogorov time scale, then a conventional mixing model can be used. But if τ_z is smaller than (or of the same order as) the Kolmogorov scale, then different modelling—based on the flamelet structure—is needed.

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W. A. Sirignano, Carnegie-Mellon University, USA. Since your two models indicate that mixing is a controlling factor in each limiting case, it might be expected that a turbulent Reynolds number dependence should be seen. Apparently, this is not allowed by assumption. Could that assumption be justified by theory or experiment?

Authors' Reply. In our theory we have assumed the Reynolds number to be high. Then it can be assumed[1,2] that the rates of mixing (i.e. dissipation rates) are determined by the larger scale turbulent motions, independent of the molecular transport properties. Thus we do not expect a Reynolds-number dependence.

In premixed turbulent combustion experiments it is difficult to vary the dimensionless parameter in a controlled manner over a significant range. But in principle the assumption could be tested by varying the (high) Reynolds number while keeping the Damkohler number fixed.

REFERENCES

1. TENNEKES, H. AND LUMLEY, J. L. (1972), *A First Course in Turbulence,* M.I.T., p. 281–2.
2. MONIN, A. S. AND YAGLOM, A. M. (1975), *Statistical Fluid Mechanics Vol. 2,* M.I.T., pp. 377–87.