Turbulent Flows
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Solution to Exercise 3.1

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In general, if $Q(U)$ is a function of U, the mean of $Q(U)$ is given by Eq.(3.20)

$$\langle Q(U)\rangle \equiv \int_{-\infty}^{\infty} Q(V) f(V)\,\mathrm{d}V. \tag{1}$$

Then, we have

$$\langle a\rangle = \int_{-\infty}^{\infty} a f(V)\,\mathrm{d}V = a\int_{-\infty}^{\infty} f(V)\,\mathrm{d}V = a, \tag{2}$$

$$\langle aQ\rangle = \int_{-\infty}^{\infty} aQ(V) f(V)\,\mathrm{d}V = a\int_{-\infty}^{\infty} Q(V) f(V)\,\mathrm{d}V = a\langle Q\rangle \tag{3}$$

and

$$\begin{aligned}\langle Q+R\rangle &= \int_{-\infty}^{\infty} (Q(V)+R(V)) f(V)\,\mathrm{d}V \\ &= \int_{-\infty}^{\infty} Q(V) f(V)\,\mathrm{d}V + \int_{-\infty}^{\infty} R(V) f(V)\,\mathrm{d}V \\ &= \langle Q\rangle + \langle R\rangle. \end{aligned} \tag{4}$$

Since $\langle Q\rangle$ and $\langle R\rangle$ are constants, by the equation (2), we get $\langle\langle Q\rangle\rangle = \langle Q\rangle$ and $\langle\langle Q\rangle\langle R\rangle\rangle = \langle Q\rangle\langle R\rangle$.

By the equations (3) and (4), the mean of the fluctuation in Q is calculated by

$$\langle q\rangle \equiv \langle Q - \langle Q\rangle\rangle = \langle Q + \langle -Q\rangle\rangle = \langle Q\rangle + \langle -Q\rangle = \langle Q\rangle - \langle Q\rangle = 0. \tag{5}$$

Since $\langle R\rangle$ is a constant, by the equations (3) and (5), $\langle q\langle R\rangle\rangle = \langle q\rangle\langle R\rangle = 0$.