Turbulent Flows
Stephen B. Pope
Cambridge University Press (2000)

Solution to Exercise 10.12

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In homogeneous turbulence, the k and ε equations become

$$\frac{dk}{dt} = \mathcal{P} - \varepsilon, \tag{1}$$

and

$$\frac{d\varepsilon}{dt} = C_{\varepsilon 1}\frac{\mathcal{P}\varepsilon}{k} - C_{\varepsilon 2}\frac{\varepsilon^2}{k}. \tag{2}$$

Consider the quatity Z defined by

$$Z = C_Z k^p \varepsilon^q. \tag{3}$$

Differentiating Eq. 3 with respect to t, we get

$$\frac{dZ}{dt} = C_Z p k^{p-1}\varepsilon^q \frac{dk}{dt} + C_Z k^p q \varepsilon^{q-1}\frac{d\varepsilon}{dt}. \tag{4}$$

Substituting Eqs.1 and 2 into 4, we get

$$\begin{aligned}\frac{dZ}{dt} &= C_Z p k^{p-1}\varepsilon^q (\mathcal{P} - \varepsilon) + C_Z k^p q \varepsilon^{q-1}\left(C_{\varepsilon 1}\frac{\mathcal{P}\varepsilon}{k} - C_{\varepsilon 2}\frac{\varepsilon^2}{k}\right) \\ &= \frac{Zp}{k}(\mathcal{P} - \varepsilon) + \frac{Zq}{\varepsilon}\left(C_{\varepsilon 1}\frac{\mathcal{P}\varepsilon}{k} - C_{\varepsilon 2}\frac{\varepsilon^2}{k}\right) \\ &= C_{Z1}\frac{Z\mathcal{P}}{k} - C_{Z2}\frac{Z\varepsilon}{k}, \end{aligned} \tag{5}$$

where

$$C_{Z1} = p + qC_{\varepsilon 1} \tag{6}$$

$$C_{Z2} = p + qC_{\varepsilon 2}. \tag{7}$$

The entries in Table 10.2 are obtained by substituting $C_{\varepsilon 1} = 1.44$, $C_{\varepsilon 2} = 1.92$ and the given values of p and q into Eqs. 6 and 7.